

Estimating Parking Time Under Batch Arrival and Dynamic Pricing Policy

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Abstract— As the urban population and the car ownership rate increase, traffic is becoming a serious urban problem. It remains difficult to reduce the large number of vehicles circling in search for free and cheaper lots in urban area. The driver's choice for on-street lots, which are generally cheaper than off-street lots but also rare, causes congestion, impact negatively the parking revenues and increases the total cost of drivers' trips. However, we propose to intervene on price as effective tool to influence driver's behavior, balance parking demand and enhance parking turnover in urban environment. This article highlights the interactions between parking price, real-time parking demand and parking time. We analyze the parking time under near-real-life conditions using the Discrete Batch Markovian Arrival Process (D-BMAP). Subsequently, we identify the optimal arrival rate that will seek to make the best use of parking resources. Finally, we propose to adopt a dynamic pricing policy that changes prices proportionally to the arrival rates on each parking and therefore reduces congestion (cruising time) and eliminates the driver's preference for some parking.

Keywords—Smart parking; Discrete-Batch Markovian Arrival process; real-time arrival rate; Dynamic price;

I. INTRODUCTION

In the last decades, the situation in the towns has completely changed. The increasing concentration of individuals and vehicle fleet pose significant challenges for urban management: Lack of parking spaces, congestion, overcrowding and pollution of the urban environment.

To tackle these challenges, urban managers should develop and implement a viable long-term strategy to deal with traffic congestion and the increased parking lot demand.

Many tools are in place and a new trend is being established in order to cope with the identified problems. However, a potential solution is the implementation of smart parking systems.

Smart parking solutions allow drivers to optimize search time, walking time at favorable prices on one side, and enable a city to optimize parking lot occupation and improve the use of parking resources.

However, as a key part of an urban development strategy, mainstreaming ICT in the mobility sectors, via IT transport solutions and mobile applications, aims to alleviate the search for parking spaces, facilitate accessibility to the different urban functions and contribute to the attractiveness and economic dynamism of the city.

In fact, the high number of vehicles circling, in urban roads, in search of parking spaces is mainly due to the lack of parking availability information and users' preference for the on-street parking lot, which is cheap compared to off-street parking which is expensive[1]. That can demonstrate two important truths: Underpriced and overcrowded on-street parking creates the urban congestion; overpriced and under-occupied off-street parking also creates economic problems, when off-street spaces remain empty, cities lose revenue.

Because, the availability and cost of a parking space are decisive factors in the driver choice to travel by private vehicle or not, or even for the choice of owning a vehicle or not; It is therefore necessary to set up a model that adjusts prices dynamically depending on real time availability of parking spaces. In this paper we propose a parking strategy that adjusts prices according to occupancy rate of parking. We highlight the interactions between dynamic parking price, real-time parking demand and parking duration to achieve the best system performance. The rest of this paper is organized as follows: The related work is presented in Section II. In Section III, the motivations to use the Discrete Batch Markovian arrival process (D-BMAP) are presented. In Section IV, We formulate the parking time under batch arrival. In Section V, simulation is illustrated and an example of a price policy that change dynamically depending on the arrival rate is presented. Last, Section VI concludes.

II. THE RELATED WORK

Smart parking may be defined as the application of advanced technologies to improve the efficiency of parking, reservation and payment processes[2]. It is based on a rather

similar architecture for the majority of existing solutions; it is managed by a set of hardware and software resources designed to process data and real-time parking information in an autonomous way.

The first smart parking solution is "Guidance"; It is mainly found in shopping center, such as at Vélizy2 [3] in France, where 3000 ParkSense were deployed in April 2010. This sort of solution allows the driver, once in the car park, to quickly locate the empty place by means of a dynamic display, at the entrance, showing the available places. This solution has evolved make possible the centralization and transmission of information to drivers before their arrival at parking by the means of mobile applications and websites [2], [4]–[9]. So that drivers can know the parking status in advance and available remotely. Although this offer make sense when the sensors are placed in streets of downtown, as the Cemavil offer [10] and the city of Nice where a smart parking infrastructure has installed in January 2013 with 1000 sensors.

Drivers, on the other hand, can take advantage of free places in car parks by downloading the dedicated mobile application. They can park on the place for as long as they wish (depending on the formula they have subscribed) and will be invoiced based on use (the difference between the time of arrival and time of departure) [13]–[16]. Some applications offer the possibility of being alerted if the authorized parking time is exceeded, or even to pay to prolong the parking time.

These new services are not only for drivers but also for owners of car parks (companies, administrations, hotels, trustees, etc.) by allowing them to rent their free spaces. The information can be transmitted to authorities.

Controlling the parking price has gained a lot of attention in recent years as an important tool for balancing the parking demand and alleviating traffic congestion.

The cities of San Francisco [11] and Los Angeles [12] have develop an interesting and effective parking system where price change dynamically; The municipality of Montreal has launched a pilot project April 2014, the idea is to adjust the parking cost according to the demand (the amount of parking spaces available per parking area) and to communicate the information with the smart phones. Instead of going around in circles, drivers will be able to see where the free parking lots are and at what price. Price will also be adjusted by sector.

The city of Casablanca is implementing a new parking policy, which will take place in three operational phases and a parallel phase. It will cover the period 2017-2023. Until 2018, the city aims to renew parking meters on the 10,000 spaces currently managed by parking meter, regulate 500 additional places, construct a new underground parking with about 500 lots and build a 250-space relay parking; 2019-2020: period of implementation of the on-street parking policy characterized by the entry into force of the control measures of spaces in the town center, as well as the construction of the car parks and a relay parking. The on-street policy includes the following measures: Adaptation of the tariff on the 10,000 parking meters of the town center, regulation of 4.000 places and set up more than 5000 places.

In the literature, the concept of smart parking has attracted a large number of researchers. In majority of papers, the parking process has been modeled by queuing theory where drivers arrive randomly to a parking according to Poisson process [17]: In [18], authors present a model to predict parking lot occupancy based on information exchanged among vehicles in the city of Brunswick, Germany. It uses a continuous-time homogeneous Markov model, where the drivers arrive according to Poisson process and the parking capacity is finite. If vehicle finds the parking full, it is rejected; This paper [19] focuses on day-time charging for plug-in electric vehicles in parking lots near commercial areas, where most vehicles have prolonged parking. For the simulation, the authors modeled the vehicles arrival as Poisson process; [20] proposed a system that helps users automatically find a free parking space at the least cost. The mathematical model for the arrival process is described by M/M/1/K/FIFO queuing model. The first "M" means that the distribution of the arrival process is Markovian (Poisson distribution), the second "M" stands for the service time distribution, which is also Markovian (exponential distribution), "1" server, and K is the number of spaces; In this paper [21], estimates the expected number for candidate parking facilities with different capacities, arrival rates, and service rates. A parking with capacity 'c' is modeled as an M/M/c/c queueing model.

But in real life, especially peak hours and urban environment, drivers may arrive in batches of different sizes, including batches of size one, from various sources to a common destination which is parking. Therefore, the arrival process encompasses complicated characteristics that can no longer be assumed to follow the conventional Poisson process. To overcome this gap, we propose using the Discrete Batch Markovian arrival process as the arrival process of vehicles to parking. This proposition, on one hand, could be as close as possible to realistic urban condition of parking behavior. On the other hand, a method for extending the capabilities of the advanced arrival process is the generalization of the existing ones [22] [36]. In our case, the best candidate to generalize the Poisson process is the Batch Markovian Arrival process.

BMAP is a subclass of Stochastic Point process that generalizes the standard Poisson Process (and other Point Processes) by allowing "batches" of arrivals, dependent interarrival times, nonexponential interarrival time distributions and correlated batch sizes [23].

III. DISCRETE BATCH MARKOVIAN ARRIVAL PROCESS (D-BMAP)

A queueing system is a system that processes items depending on the processing order or the discipline adopted. The words "item" and "processing" are generic terms. An "item" could refer to customers [24]. In our case, the customers are vehicles arriving at a parking system (queue). After a parking time (waiting time) they pay (service) and leave the system (depart).

Queueing systems with batch service have been the subject of large number of researches. However, the principal aim is to correct the gap caused by the models considering an

uncorrelated arrival process, which is unrealistic in several real-life situations.

The analysis of D - $BMAP/G/1$ queue have been studied in the literature [25]–[30].

In this paper, we consider a D - $BMAP/G/1/N$ - ROS queueing system with a single node queue and finite capacity. The customers arriving according to Discrete Batch Markovian arrival process (D - $BMAP$) and are selected for service in random order (ROS) according to their parking time. This system is also called a loss system. The capacity (number of spaces) of parking lot is finite and the queue has the same capacity as the parking lot.

The customers arrive in batches of random size and the capacity is finite. However, two strategies arise if the batch size is larger than the available capacity [31]:

- Partial-batch acceptance strategy (PBAS): part of the batch is accepted and the rest is rejected;
- Whole-batch acceptance strategy (WBAS): the whole batch is rejected

Here, we assume that the admission strategy for a batch is PBAS. This means that the total number of vehicles in the parking does not exceed N . Vehicles arriving when the queue is full are lost.

The distribution of the number of customer arrivals per slot depends on parking access point number, parking location (on street or off street lots), time of day (peak hours), ...Etc.

In Discrete Batch Markovian arrival process, the arrivals are governed by an underlying m -state Markov chain having probability d_{ij}^n , $1 \leq i, j \leq m$, $n \geq 0$, with a transition from state i to state j and a batch arrival of size n . Here $D_0 = (d_{ij}^0)$ is an $m \times m$ non-negative matrix which governs no arrival, whereas $D_n = (d_{ij}^n)$, $n \geq 1$ are $m \times m$ non-negative matrices which govern a batch arrival of size n . The matrix $D = (d_{ij}) = \sum_{k=0}^{\infty} D_k$ with $D\mathbf{e} = \mathbf{e}$, where \mathbf{e} is a column vector of ones with an appropriate dimension, is a stochastic matrix corresponding to an irreducible Markov chain underlying D - $BMAP$. We call the actual state of this chain the ‘‘phase’’ of the arrival process. Let $\bar{\pi}$ be the $1 \times m$ stationary vector of the underlying Markov chain (UMC), i.e., $\bar{\pi}D = \bar{\pi}$; $\bar{\pi}\mathbf{e} = 1$. The fundamental arrival rate (the average number of customer arrivals per slot) of this process is given by $\lambda^* = \bar{\pi} \sum_{k=0}^{\infty} k D_k \mathbf{e}$ and the average number of batch arrivals per time unity is $\lambda_g = \bar{\pi} \hat{D}_1 \mathbf{e}$, where $\hat{D}_n = (\hat{d}_{ij}^n) = \sum_{k=1}^{\infty} D_k$, $n \geq 1$.

Let the time axis is divided into fixed-length intervals $0, 1, 2, \dots, t, \dots$. Here we discuss the models for late arrival system with delayed access (LAS-DA) and therefore, a potential batch arrival takes place in $(t-, t)$ and a potential departure occurs in $(t, t+)$; see [24].

IV. MODEL DESCRIPTION

Assume that vehicles arrive at a parking lot (queue) with batch of different sizes, after a random parking time, costumers must report to the cashier to pay parking fees and leave the system.

The objective of this paper is to limit the parking time at certain periods or in certain areas and influencing driver’s behavior using a dynamic pricing policy.

Parking price represents a powerful tool way to control the parking demand by encouraging customers to reduce or delay their demand for parking from peak periods to off-peak periods, improve the vehicle turnover at certain periods or in certain areas and balance the parking demand among different parking lots.

Thus, Prices vary proportionally with the parking occupancy rate which is generally estimated from the sensors data historical or ticket vending machines (parking meters). The price variation aims to improve the performance indicators of parking such as turnover, waiting time and the service time.

In the following, we shall identify the optimal arrival rate λ_g that guaranty an efficient and balanced system with finite capacity. We propose to analyze the parking system under D - $BMAP$, by simulating the arrival rate and the system average load (the average number of customers in the system) given a parking time (waiting time) and a finite parking capacity.

To calculate the waiting time, it is required to first calculate the blocking probability as we deal with a finite capacity system. We define:

- The waiting time as the time interval from the instant that a vehicle enters the parking (queue), to the instant that the vehicle head to payment service.
- The blocking probability, or loss probability, as the probability that an arriving vehicle finds the parking full.

It should be noted that, to set up such a system, it is necessary to obtain the real time parking lots supply and demand data (parking information). This information is provided by detection technologies (arrival rate), on-line reservation data base (reservation request) or ticket vending machines.

A. Blocking probability:

Here we obtain the blocking probability of an arbitrary customer in a batch. Let PBLA be the probability that an arbitrary customer in a batch are lost upon arrival.

An arbitrary customer in a batch is lost if he finds n ($0 \leq n \leq N$) customers in the queue upon arrival and his position in his batch is $\geq N + 1 - n$:

$$PBLA = v(0) \sum_{k=N+1}^{\infty} H_k \mathbf{e} + \sum_{n=1}^N \{\pi(n)\} \sum_{k=N+1-n}^{\infty} H_k \mathbf{e}$$

Equation1. The blocking probability of an arbitrary customer.

Where:

$\mathbf{H}_k = \frac{\hat{D}_k}{\lambda^*}$, ($k = 1, 2, 3, \dots$) is the matrix of order $m \times m$ whose element $[\mathbf{H}_k]_{ij}$ represents the probability that the position of an arbitrary customer in an arriving batch is k with phase changes from state i to j and $\hat{D}_k = \sum_{n=k}^{\infty} D_n$, $n \geq 1$.

B. Waiting time analysis:

In this section, we obtain the waiting time of an arbitrary customer of an accepted batch in the queue under the random order service (ROS) queueing discipline.

It should be noted that the waiting time of a customer under FIFO queueing discipline should be calculated considering its position of in the queue. The formulas below show how to calculate the waiting time of and an arbitrary W_A customer in the queue under FIFO queueing discipline:

$$W_A^*(z) = \frac{1}{(1 - PBL_A)} \left[v(0) \sum_{k=1}^{N-1} H_k \{S(z)\}^{k-1} + \sum_{n=0}^{N-1} \pi^*(n, z) \sum_{k=1}^{N-n} H_k \{S(z)\}^{n+k-1} \right]$$

Equation 2. The vector-generating function (v.g.f.) of the actual waiting time of an arbitrary customer in a batch under FIFO discipline.

Suppose first customer of an arrival batch finds the server busy with n customers are waiting in the queue and one customer is taking service with some remaining service time is there. $\pi^*(n, z)$: represents remaining service time of the customer who is taking service; $\{S(z)\}^n$: denotes service times of n customers. $v(0)$: When a batch arrives and finds the parking empty.

The waiting time (W_A) is obtained by differentiating $W_A^*(z)$ and then substitute $z = 1$. For more details see [31].

However, in ROS discipline, the selection of a customer (finishes parking time and goes to payment service) does not depend on position of the customer in the queue as well as his position in the arrived batch. In this discipline, any waiting customer can be selected for service with equal probability. That is, if there are $n (\geq 1)$ number of customers in the system at a service completion epoch then each customer has an equal chance to get selected for service with probability $1/n$.

In addition to that in a batch arrival queueing system, $l (\geq 1)$ number of customers may arrive in the system during the service time of a customer. Hence, a customer is selected for the next service with probability $1/(n+l-1)$. This process continues, i.e, a customer is selected randomly for service and during the service time, new batches of variable sizes may arrive in the system.

In such a way, we can calculate the waiting time as following:

$\frac{\bar{\pi} n D_n e}{\lambda^*}$, ($n = 1, 2, 3 \dots$): Is the probability that an arbitrary customer belongs to a batch of size n .

$\frac{a_{ij}^n}{\bar{\pi} D_n e}$, ($n = 1, 2, 3 \dots$): Is the probability that a batch arrival consisting of n customers occurs and phase changes from state i to j .

$$W = \frac{1}{(1 - PBL_A) \lambda^*} \left[\sum_{n=1}^N v(0) D_n + \sum_{k=0}^{N-1} \sum_{n=1}^{N-k} \left(\frac{n}{k+n-1} \right) \pi(n) D_n \right]$$

Equation 3. Waiting time of an arbitrary customer in a batch under ROS discipline.

Where: n is the batch size.

But in parking area, the probability that a customer is selected for service depends on his parking duration i.e, a customer leaves after a random parking duration. Consequently, the waiting time can be calculated as:

$$W = \frac{1}{(1 - PBL_A) \lambda^*} \left[\sum_{n=1}^N v(0) D_n \xi(T_p = \tau) + \sum_{k=1}^{N-1} \sum_{n=1}^{N-k} \left(\frac{n}{k+n-1} \right) \pi(k) D_n \xi(T_p = \tau) \right]$$

Equation 4. Waiting time of an arbitrary customer in a batch under ROS discipline in parking application.

Where:

$\xi(T_p = \tau) = 1 - e^{-\lambda \tau}$: is the probability that the customer stays in the parking τ time.

V. SIMULATION AND PRICING STRATEGY

A. Model simulation:

We consider a parking lot of a university campus with 10 places. We assume that customers arrive at specific times (8h, 10h ... etc.). Therefore, we will consider the uniform case where all customers come at the same time and have the same chance of leaving after a parking time.

The arrival rate λ^* is expressed as a function of the waiting time from equation 3. The aim is to find the optimal arrival rate λ_g to park for a certain parking time (waiting time in the queue).

The simulations were performed on MATLAB.

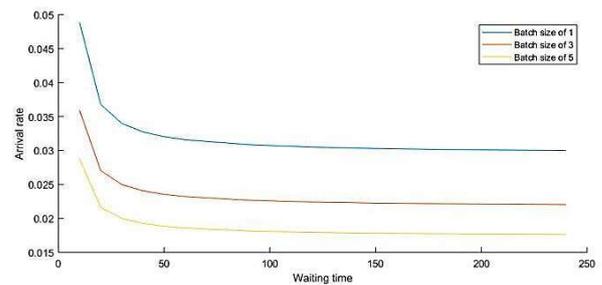


Figure 1. The optimal arrival rate corresponding to each parking time with different batch size.

The objective of this work is to control the parking time by means of pricing policy. Thus, the arrival rate corresponding to a parking time should not exceed an optimal value (λ_g). If the optimal value of the arrival rate is exceeded, the parking price should increase in order to reduce the parking demand and improve the vehicle turnover.

B. Average Load:

Using Little's rule, we can obtain the average Load in the queue (Parking):

$$[L] = \lambda' * W$$

Where:

$[L]$: Is the expected average queue-length (the expected average number of vehicles in the parking);

$\lambda' = \lambda * (1 - PBL_A)$: Is the effective arrival rate;

W : is the waiting time or the parking time.

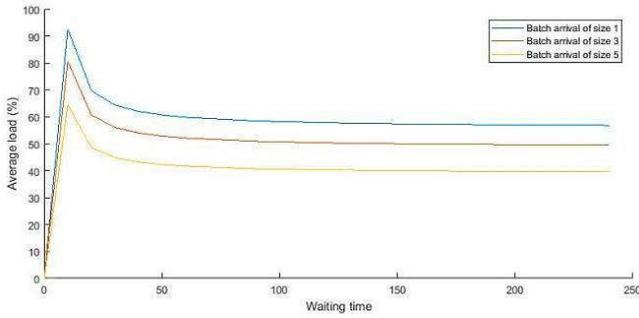


Figure 2. The average load of parking.

The Fig.2 simulates the expected average number of vehicles in the parking considering different values of arrival rates λ_g . The average values vary between 50% and 70% with respect to λ_{gopt} values.

In order to keep these performances a pricing policy that adjusts prices to achieve.

C. Dynamic price:

In urban areas, drivers prefer the cheapest parking spaces. Obviously, they all head to the on-street parking lots, causing congestion and leaving the off street with great availability due to their price which is expensive. However, Parking pricing strategies are important tools for balancing the parking lots demand between on street and off-street parking lots in urban areas.

We propose to intervene on prices of on-street lots. Because a significant percentage of drivers prefer to use the off-street parking lot even if the price is expensive (because of their social status, security, ... etc.). So, we proposed a strategy that changes the price of the on-street parking lot once detecting that the arrival rate exceeds its optimal value. We keep track of the on-street and off-street arrival rates to make decision on price. For example, in university with 10 parking lots, we find that the average parking time is around 4 hours in the morning and 4 hours in the afternoon (because of the work schedule). In the simulation results, we can see that the optimal value of arrival rate that should not be exceeded is: $\lambda_g=0,03$ for $n=1$, $\lambda_g=0.026$ for $n=2$ and $\lambda_g=0.022$ for $n=3$. If the optimal value of the arrival rate is exceeded, the price can be increased

or doubled to guarantee a balance of the system, and the drivers will have to look in other car parks.

TABLE I. PRICES CHANGE DYNAMICALLY DEPENDING ON THE ARRIVAL RATE FOR A PARKING TIME OF 4 HOURS

Arrival rate	$\lambda_{gopt}/4$	$\lambda_{gopt}/2$	λ_{gopt}	$> \lambda_{gopt}$
Price	$P_{on-street}$	$60\% * P_{off-street}$	$100\% * P_{off-street}$	$120\% * P_{off-street}$

λ_{gopt} : the optimal value of arrival rate that should not be exceeded;

$P_{on-street}$: initial price of on-street parking lot;

$P_{off-street}$: price of off-street parking lot.

To generalize, we use the same assumptions to balance the use of on-street and off-street parking lots. When detecting the optimal value of the arrival rate is exceeded, we keep the price of off-street parking lot fixe and change that of the on-street until they become equal:

$$\lambda_g(\text{Arrival rate}) = \beta * \lambda_{gopt} \quad \text{Where } \beta \in [0,1]$$

$$P_{on_street} = \beta * P_{off_street}$$

Table 1 illustrates an example of Prices dynamically change depending on the arrival rate for a parking time of 4 hours. In this way, the driver does not give any preference to a type of places, and a balance will be guaranteed.

VI. CONCLUSION

To address these urban parking problems a policy that adjusts prices dynamically to achieve. We keep track of real-time arrival rates and change the parking price proportionally. This policy allows an effective parking access and space utilization. This paper highlights the interactions between dynamic parking prices, real-time demand for parking and parking time to achieve the best performance. We first estimate the parking time under the realistic conditions using the discrete Batch Markovian arrival process (D-BMAP). We got the optimal arrival rate to park in a parking lot for a parking time. By exploiting this result, we proposed a dynamic parking pricing schemes that produces optimal parking arrival rates and reduces congestion (Cruising time) and the problem of preferred parking.

In the future, we aim to evaluate our system using real-time data of the city of Casablanca in Morocco.

It would also be useful to simulate our model in these areas with different parking arrival scenarios in real life.

Acknowledgment

The Foundation for Research, Development, Innovation and Engineering Sciences (FRDISI) has provided financial support for our research.

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